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PLANE EVOLUTION OF A SATELLITE'S ELLIPTICAL ORBIT UNDER THE INFLUENCE OF A LATERAL DISTURBING FORCE

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ABSTRACT. An investigation is made of satellite motion as a mass point under the influence of disturbing acceleration, constant in modulus and directed normal to the osculating orbital plane. The shape of the orbit is always constant and the problem is reduced to a study of a system of three equations defining the precession of the orbital plane.

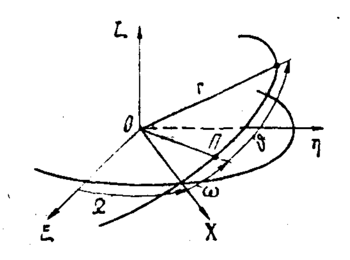
The author investigates satellite motion as a mass point under the influence of disturbing acceleration, constant in modulus and directed normal to the osculating plane of the orbit. Under the influence of such disturbing forces, the shape of the orbit at every instant of time will remain constant [1]. In this case, the problem is reduced to an investigation of a system of three equations defining the precession of the orbital plane (Equations (1.2) and (1.3) in the work [2]). /36*

$$\frac{di}{dv} = \varepsilon \frac{\cos(\vartheta + \omega)}{(1 + e\cos\vartheta)^3} \qquad \frac{d\Omega}{dv} = \varepsilon \frac{\sin(\vartheta + \omega)}{\sin i\,(1 + e\cos\vartheta)^3}$$

$$\frac{d\omega}{dv} = -\varepsilon \frac{\cot i \sin(\vartheta + \omega)}{(1 + e\cos\vartheta)^3} \tag{1}$$

$$\left(\varepsilon = \frac{j}{g(p)},\ e = \text{const},\ p = \text{const}\right)$$

Here i is the inclination of the orbit, ω is the angular distance of the perigee from the node, Ω is the longitude of the ascending node, ϑ is the true anomaly (figure), e is orbit eccentricity, j is the modulus of the disturbing acceleration, $g(p)$ is gravitational acceleration, corresponding to the radial distance of the satellite, equal to the orbit parameter p; the angular variable v is a monotonically rising time function [2].



Let us examine a case when the ratio $j/g(p)$ is small, and take the constant ε as the small parameter of the problem.

Without limiting the universality, the system of $\xi\eta\zeta$ are distributed such that the axis ζ is directed along the lines of apsides of the undisturbed orbit. The following initial values of the angles

*Numbers given in the margin indicate pagination in original foreign text.

will correspond to the above distribution of inertial axes:

$$i_0 = 90^\circ, \quad \omega_0 = \pm 90^\circ \tag{2}$$

The initial value Ω_0 is arbitrary (for definiteness it is assumed that $\Omega_0 = 0$).

The averaging principle of N. M. Krylov and N. N. Bogolyubov [3] is applied to Equations (1), which have the so-called standard form. Averaging by ϑ, we obtain

$$\frac{di}{dv} = \varepsilon J \cos\omega, \quad \frac{d\omega}{dv} = -\varepsilon J \sin\omega \cot i, \quad \frac{d\Omega}{dv} = \varepsilon J \frac{\sin\omega}{\sin i} \quad \left(J = -\frac{3}{2}\frac{c}{(1-e^2)^{3/2}}\right) \tag{3}$$

It is easy to see that the solution

$$i \equiv 90^\circ, \quad \omega \equiv 90^\circ, \quad \Omega = \varepsilon J v \tag{4}$$

corresponds to the initial values (2).

Solution (4), which has such a simple form owing to the selected distribution of inertial axes, shows that the influence of the investigated disturbing acceleration leads to the rotation of the plane of the elliptical orbit relative to the line of apsides which remains fixed in inertial space. It is characteristic that the orbital plane will evolve in such manner irrespective of the point of /37 the primary undisturbed orbit at which the disturbing acceleration starts to affect the satellite (i.e., independent of the initial value of the true anomaly ϑ_0).

In the general case, during the rotation of the orbital plane relative to the fixed line of apsides, having an arbitrary orientation in the inertial axes, the change of the angles i, ω, and Ω will be described for the formulas

[Note: tg = tan]

$$\cos i = \sqrt{1-E^2}\sin(\varepsilon J v + C_1), \quad \sin i \sin\omega = E$$
$$\Omega = \operatorname{arc\,tg}\{E\ \operatorname{tg}(\varepsilon J v + C_1)\} + C_2 \tag{5}$$

Here E, C_1, and C_2 are random constants which are expressed through the initial values of the angles i_0, ω_0, and Ω_0. The important constant among the three is E which represents the sinus of the angle formed by the line of apsides with the inertial plane $\xi\eta$.

Thus, with E = 1 solution (4) follows from the general solution (5). With E = 0 we obtain one more simple solution

$$i = i_0 + \varepsilon J v, \quad \omega \equiv 0, \quad \Omega \equiv \text{const} \tag{6}$$

which corresponds to the rotation of the orbital plane relative to the line of apsides lying in the inertial plane $\xi\eta$ and coinciding with the fixed node lines.

Therefore, on the basis of solutions of averaged equations, it may be concluded that, to an accuracy of small oscillating terms, the evolution of a satellite's orbit in the present problem is reduced to a "pure" rotation of the orbital plane relative to the fixed line of apsides.

Note that this problem has been examined in the past (see the work [4]), however, the present geometrical interpretation of the solution was not known.

As an example, the approximate solutions (4) and (6) were compared with the corresponding explicit solutions computed on an electronic computer. The comparison shows good agreement between the two in the intervals confined by the boundaries of applicability of the averaging method. For instance, with $\varepsilon = 0.01$, these intervals have an order of hundreds of satellite revolutions.

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